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MODELLING OF PLATE-TYPE ACOUSTIC METAMATERIALS USING ANSYS

ABSTRACT

The acoustic metamaterials are frequently designed using FEA techniques. This paper presents the detailed method of employing the periodic boundary condition and Floquet periodic boundary condition in the FEA software – ANSYS 2022 R1. The procedure operates on APDL (Ansys Parametric Design Language) code inserted into analysis in Mechanical Application. An example of plate-type metamaterial with antisymmetric periodicity is used for verification of the procedure.

KEYWORDS: FEM, acoustic metamaterials, periodicity, periodic boundary condition, Floquet periodicity

1. INTRODUCTION

Acoustic metamaterials (AMMs) are artificial periodic structures, which enable control of the acoustic field to a level not reachable by conventional methods. The 2D structures, such as plate-type AMMs, received growing attention over the last decades, especially in noise control. Locally resonances forming in the given grid lead to band gaps development and hence, break the law mass for acoustic insulation. This advantage is attainable only for a finite frequency range. Therefore, acoustic metamaterials may be employed to control tonal and narrow-banded noise or in applications with weight and space limitations.

The design process of AMMs frequently operates on Finite Element Analysis (FEA). FEA joins the usage simplicity with the ability to compute the solutions for complex geometrical objects. The possible analysis types which could estimate the AMMs features are Linear Dynamic Analysis (e.g., Modal and Harmonic Response) or Acoustic Analyses (e.g., Harmonic Acoustics). All mentioned types are the issue of this paper.

The single metamaterial design consists of a dense grid of periodically distributed cells. Each cell may contain thousands of mesh elements thus only practical method to solve the results is to use the periodic boundary condition or Floquet periodic boundary condition on a single cell.

2. MATERIALS

The model sample employed to present the following analysis will be low-band plate-type metamaterial as in Fig. 1. The resonant structures are beams fixed rigidly in the middle with a holder to the flat panel. The whole plate is periodic in the XY plane and excited by an incident plane wave along the Z axis. The periodicity in the given material is odd (marked in Fig 1 with dashed lines), which means that along the axis X, the cell is inverted through the axis Y and vice versa. The four adjacent cells (marked with blue in Fig 1) assemble a larger partition with regular periodicity.

Table 1. presents all material parameters and dimensions implemented in the model. The materials utilised in simulations are orthotropic (cardboard for beams and holders and paper honeycomb for the base panel).

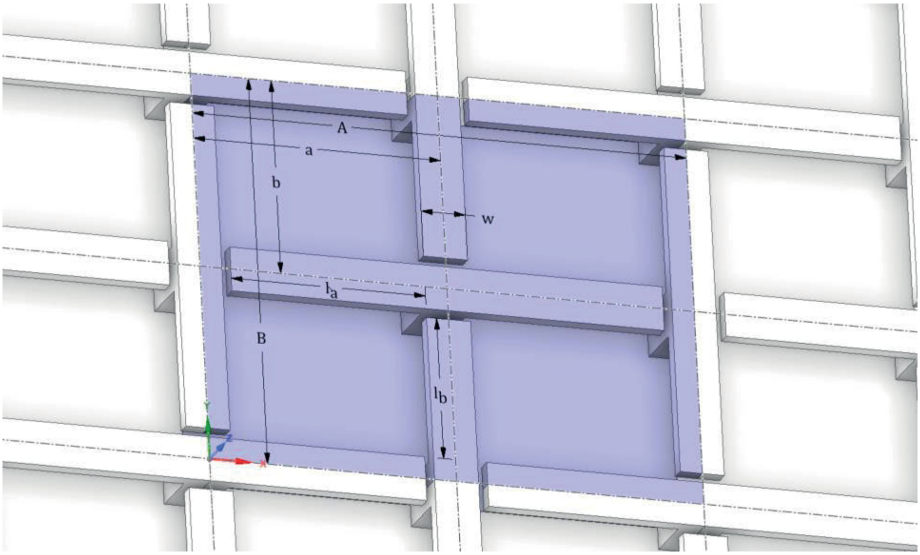

Fig. 1 The simulated plate-type metamaterial scheme. The single cell is marked with colour blue. With antiperiodic cells are marked with dashed lines.

Table 1 Material parameters and structure dimensions used in simulation

<table>
<tr><th></th><th colspan="3">Material parameters</th><th colspan="3">Structure dimensions</th></tr>
<tr><th></th><th>Parameter</th><th>Direction</th><th>Value</th><th>Dimension</th><th>Direction</th><th>Value</th></tr>
<tr><td rowspan="9">Beams and holders</td><td rowspan="2">Young Modulus</td><td>y & z</td><td>1,02 GPa</td><td rowspan="2">Regular lattice constants A</td><td>x (A)</td><td>112 mm</td></tr>
<tr><td>x</td><td>2,05 GPa</td><td>y ()</td><td>88 mm</td></tr>
<tr><td rowspan="2">Poisson’s ratio</td><td rowspan="2">xy, xz & yz</td><td rowspan="2">0,3</td><td rowspan="2">Antiperiodic lattice constants</td><td>x ()</td><td>56 mm</td></tr>
<tr><td>y ()</td><td>44 mm</td></tr>
<tr><td rowspan="2">Material dependent damping ratio</td><td rowspan="2">x, y & z</td><td rowspan="2">0,0344</td><td rowspan="2">Beam length</td><td>x ()</td><td>44 mm</td></tr>
<tr><td>y ()</td><td>32 mm</td></tr>
<tr><td rowspan="3">Density</td><td rowspan="3">–</td><td rowspan="3">667 kg/m^3</td><td>Beam thickness</td><td>x, y</td><td>3 mm</td></tr>
<tr><td>Beam width</td><td>x, y</td><td>10 mm</td></tr>
<tr><td>Holder dimensions</td><td>x, y</td><td>10 x 10 x 3 mm</td></tr>
<tr><td>Base panel</td><td>Young Modulus</td><td>x, y & z</td><td>0,11 GPa</td><td>Panel thickness</td><td>–</td><td>10 mm</td></tr>
</table>

3. METHODS

The basic analyses needed to simulate the plate-type acoustic metamaterials are:

- Linear Dynamic Analysis - to calculate the results only concerning AMMs body, e.g., Modal Analysis to obtain modes and dispersion curve or Harmonic Response Analysis to obtain the vibration response to known excitation,
- Acoustic Analyses, to calculate the AMM behaviour coupled with the surrounding acoustic field (*Acoustic Region*) e.g., Harmonic Acoustics to calculate the Transmission Loss.

To limit the size of the periodic model, the periodic Boundary Condition should be applied. In Ansys Mechanical Application there is a tool enabling such condition – the Linear Periodic Symmetry Region. This tool specifies the constraint equations linking the degrees of freedom (DOFs) of the cells' low to the high boundary (Fig. 2). Unfortunately, its application is restricted to structural and thermal or thermal-electric analyses. Furthermore, the symmetry is viable only in a single direction, preventing its implementation in 2D AMM. [1]

The concept will be recreated in terms of APDL Commands and executed within every performed analysis.

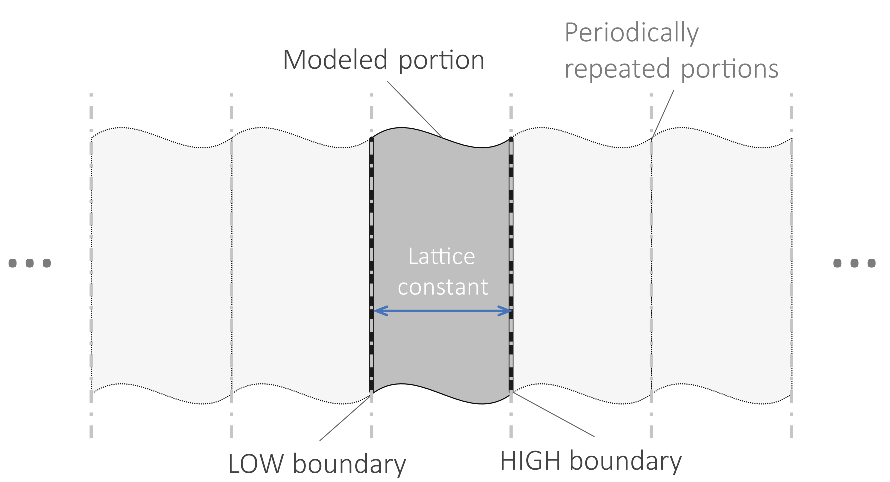

Fig. 2 Linear Periodic Symmetry in one direction

3.1. PERIODIC BOUNDARY CONDITION

Let us consider the model illustrated in Fig 3. Every marginal surface belongs to one of the low or high boundaries in both directions:

- Low X/Y – name selection for low X/Y boundary,
- High X/Y – name selection for high X/Y boundary.

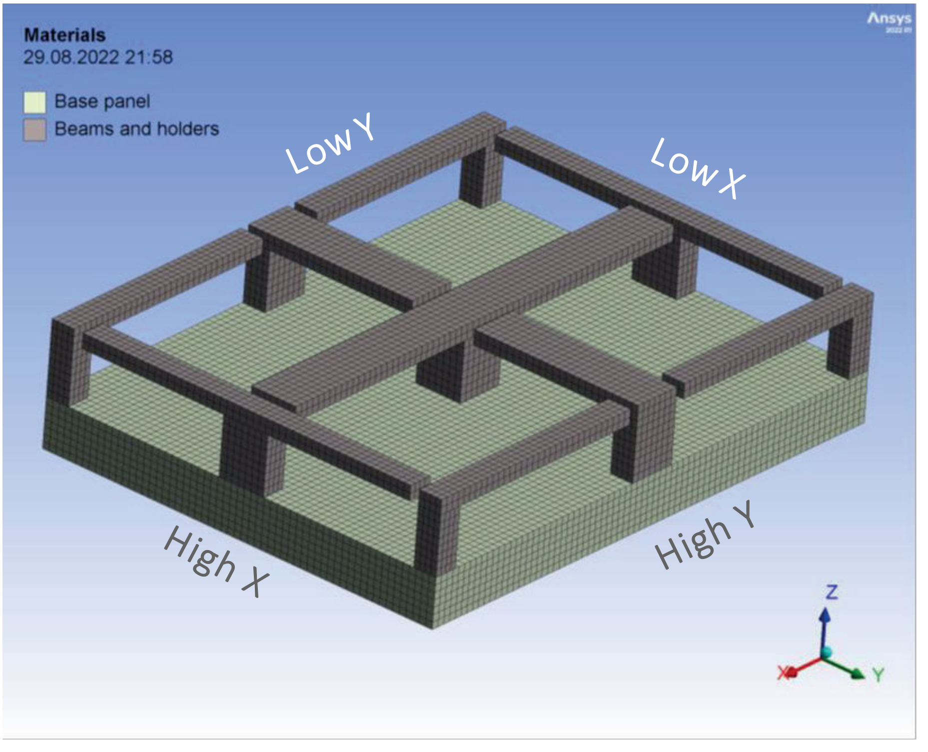

Fig. 3 Exemplary model with generated mesh for regular lattice

The mesh on each pair of boundary surfaces must be identical. The bottom surface of the base panel is fixed and every connection between the bodies is set to "bonded".

The prepared model and mesh enable to execute the APDL Commands.

The valid constrain equation requires the definition of pilot nodes for both directions (`Pilot_X/Y`). Each pilot node is located in the invariable point. Pilots should be assigned to mass-type elements with the weight of zero and DOFs specified exclusively to displacements (`KEOPT(3)` of `MASS21` element set to 2). Correspondingly, pilots must have separate materials and real constants. This condition could be defined by the means of APDL.

The easiest method to constrain the nodes is to define corresponding matrices for both low and high boundaries. This is accomplished by defining the matrix with every node index number and location for the low boundary followed by building the complementary matrix for the high boundary. The second may be implemented by searching each node by its location respectively to the opposite one. The commands for X direction are:

```
nsel,s,LOC,X,(Low_X(ii,2)-tol_X)+shift_X,(Low_X(ii,2)+tol_X)
+shift_X
nsel,r,LOC,Y,Low_X(ii,3)-tol_X,Low_X(ii,3)+tol_X
nsel,r,LOC,Z,Low_X(ii,4)-tol_X,Low_X(ii,4)+tol_X
*get,High_X(ii,1),node,,num,max
```

where:

`Low/High_X` – the matrix of low/high boundary, the row number refers to node index and column number to data type: 1 – node index, 2, 3, 4 – locations in X, Y and Z directions respectively;
`tol_X/Y` – toleration of node location – with the value of approximately 1/10 of the minimal element size;
`shift_X/Y` – lattice constants: *A* or *B*.

For the Y direction, commands are congruent.

For antisymmetric geometry (Fig 4), as in the examined example, the Y-component position for X direction periodicity and X-component position for Y direction periodicity should be reversed, hence the code lines should be replaced with:

```
nsel,r,LOC,Y,shift_Y-Low_X(ii,3)-tol_X,shift_Y-Low_X(ii,3)
+tol_X
nsel,r,LOC,Y,shift_X-Low_Y(ii,2)-tol_Y,shift_Y-Low_Y(ii,2)
+tol_Y
```

Finally, the symmetric nodes are in relation according to equation linking their DOFs. For Linear Dynamic Analysis the only DOFs linked are displacements:

$$\mathbf{u_H} = \mathbf{u_L} + \mathbf{u_P} \tag{1}$$

where $\mathbf{u_H}$, $\mathbf{u_L}$, $\mathbf{u_P}$ are nodes displacement vectors.

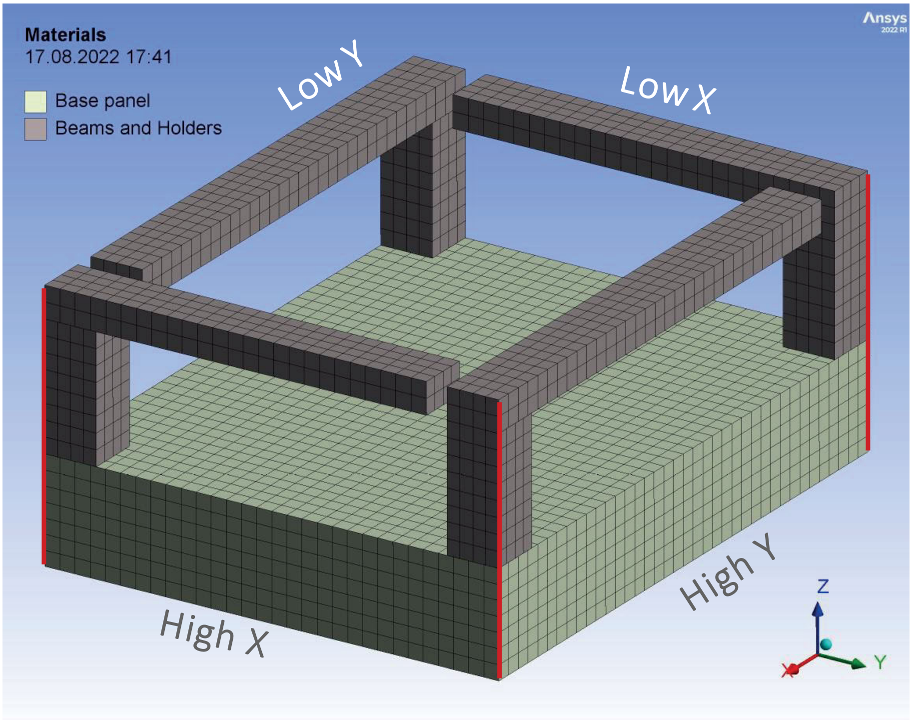

Fig. 4 Exemplary model with generated mesh for antiperiodic lattice

The APDL implementation (in example of X direction) for regular periodic lattice is as follows:

```
CE,NEXT,,Low_X(ii,1),UX,-1,High_X(ii,1),UX,1,Pilot_X,UX,1
CE,NEXT,,Low_X(ii,1),UY,-1,High_X(ii,1),UY,1,Pilot_X,UY,1
CE,NEXT,,Low_X(ii,1),UZ,-1,High_X(ii,1),UZ,1,Pilot_X,UZ,1
```

For the Y direction, commands are congruent.

For antiperiodic lattice, the Y-component displacements for X direction of periodicity and the X-component displacements for Y direction of periodicity are reversed, which replaces the respective lines with:

```
CE,NEXT,,Low_X(ii,1),UY,-1,High_X(ii,1),UY,-1,Pilot_X,UY,1
CE,NEXT,,Low_Y(ii,1),UX,-1,High_Y(ii,1),UX,-1,Pilot_Y,UX,1
```

In the case of Acoustic Analyses, the additional equation links the acoustic pressure for surrounding medium (*Acoustic Region*):

$$p_H = p_L \tag{2}$$

where p_H, p_L, p_P are nodes acoustic pressures.

The APDL command (in example of X direction) is given only for single constraint equation. Since the quantity is scalar, there is no need to diversify the relation for antisymmetry:

```
CE,NEXT,,Low_X(ii,1),PRES,-1,High_X(ii,1),PRES,1
```

The above procedure for antiperiodic lattice is *not* precise. Reversing the axes of location and displacement twice for x and y directions leads to significant error. The X and Y displacement components on the marginal edges (marked with red lines in Fig. 4) are falsely fixed. The explanation is, that the diagonal nodes of low and high edges have linked displacements in simultaneously +X, +Y and -X, -Y directions. The consequence of the error is explained in paragraph 4.

3.2. FLOQUET PERIODIC BOUNDARY CONDITION

The dispersion curve relates the frequency of a wave propagating through the medium to its wave number. The homogeneous body dispersion curve is linear for any given wave frequency, whereas in metamaterials the band gaps could form. The width and height of a band gap may be crucial conditions in the metamaterial project process. The dispersion relation is the solution of the Bloch-Floquet theorem [2], [3]:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}(\mathbf{x}_0)e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}_0)} \tag{3}$$

where: $\mathbf{u}(\mathbf{x}),\ \mathbf{u}(\mathbf{x}_0).$ are the displacement vectors in the respective locations, and $\mathbf{k}$ is the wave vector (Floquet wavenumber).

The relation in examined model will adopt the form of:

$$\mathbf{u}_\mathbf{H} = \mathbf{u}_\mathbf{L}e^{i\mathbf{k}r} \tag{4}$$

where r stands for lattice constant (A or a for X direction and B or b for Y direction).

Such relation is the eigenvalue problem with defined values of $\mathbf{k}$, with the results obtainable in Ansys through the Modal Analysis. Prior to the analysis, the $\mathbf{k}$ vector must be specified. The $\mathbf{k}$ vector should cover the whole indivisible Brillouin Zone. In a given 2D model with perpendicular lattice, the Brillouin Zone with every possible $\mathbf{k}$ vector is as in Fig 5. This means that for each $\mathbf{k}$ value in directions x and y, starting from point Γ through X, M to Γ, an individual Modal Analysis is conducted. k equal to 0 simplifies the equation (4) to the eq. (1), which makes this a simple Modal Analysis as in paragraph 3.1.

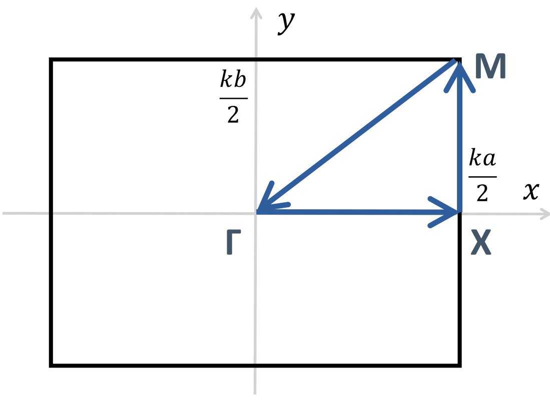

Fig. 5 Brillouin zone for perpendicular lattice

After expanding equation (4), the complex relation emerges:

$$\begin{bmatrix} \mathbf{u}_H^R \\ \mathbf{u}_H^I \end{bmatrix} = \begin{bmatrix} \cos ka & -\sin ka \\ \sin ka & \cos ka \end{bmatrix} \begin{bmatrix} \mathbf{u}_L^R \\ \mathbf{u}_L^I \end{bmatrix} \tag{5}$$

The imaginary displacement field $\mathbf{u}^I$ requires duplicating the mesh as in the Fig. 6. [2].

The APDL implementation of constraint equations based on eq. (4) are as follows.

For real high boundary, the periodicity in X direction for each node is represented by:

```
CE,NEXT,,Low_RX(ii,1),U,-COS(K_X*shift_X),High_RX(ii,1),U,-1,…
 … Low_IX(ii,1),U,SIN(K_X*shift_X)
CE,HIGH,,Pilot_X,U,1
```

Where `U` stands for `UX`, `UY` and `UZ` displacements and `K_X` is wave number in X direction.

The lines for Y direction are corresponding:

```
CE,NEXT,,Low_RY(ii,1),U,-COS(K_Y*shift_Y),High_RY(ii,1),U,1,…
 … Low_IY(ii,1),U,SIN(K_Y*shift_Y)
CE,HIGH,,Pilot_Y,U,1
```

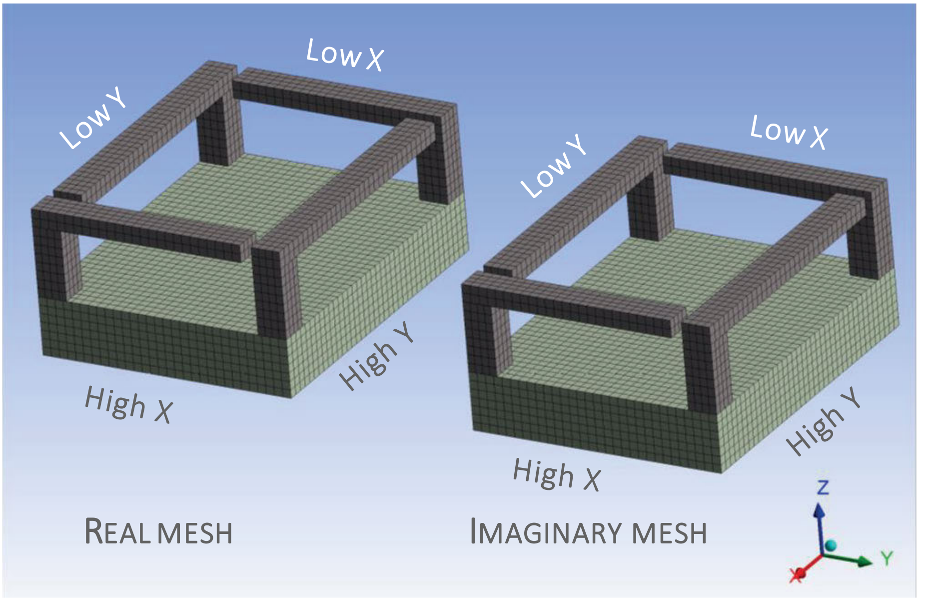

Fig. 6 Real and imaginary mesh — the antiperiodic lattice example

For the imaginary high boundary, the lines are:

```
CE,NEXT,,Low_RX(ii,1),U,-SIN(K_X*shift_X),High_IX(ii,1),U,-1,…
… Low_IX(ii,1),U,-COS(K_X*shift_X)
CE,HIGH,,Pilot_X,UZ,1
CE,NEXT,,Low_RY(ii,1),U,-SIN(K_Y*shift_Y),High_IY(ii,1),U,1,…
… Low_IY(ii,1),U,-COS(K_Y*shift_Y)
CE,HIGH,,Pilot_Y,U,1
```

4. RESULTS AND DISCUSSION

The procedure was assessed by the Modal and Mode-Superposition Harmonic Response Analyses. The Modal Analysis was prepared as described in paragraph 3, while Harmonic Response Analysis had an acceleration load of 1 mm/s^2 applied to the fixed surface. The response spectra of vibration velocity level were averaged from displacements in the Z direction for nodes located near the free end of the beam.

The verification of the Linear Dynamic Analyses results confirms that the procedure links the low boundary surfaces to high boundary surfaces properly. The mode shape for both directions is compatible with its representations on the opposite boundary (Fig 7). Unfortunately, the falsely fixed edges remove from the results modes with non-zero displacements. Table 2 represents the frequencies of identically shaped modes for both lattices – modes 1 and 3 are not perceptible in the antiperiodic lattice. Nevertheless, in specific circumstances, the procedure gives a partial solution with valuable information. When the flat structure operates on an acoustic wave perpendicular to itself (which is often the case in plate-type AMMs), it is excited in the Z direction. If the local resonators are symmetrical to the mentioned supports and the supports are located on the cell edges, the max-

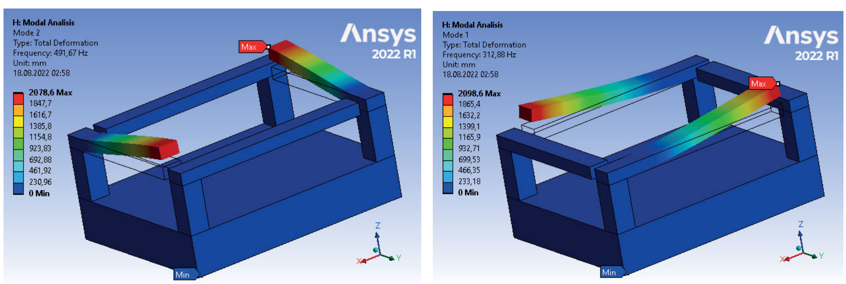

Fig. 7 Total deformation of primary modes in Z direction for antiperiodic lattice

Table 2 Modes for antiperiodic and regular lattice

Mode number	**Antiperiodic lattice mode [Hz]** *partial solution*	**Regular lattice modes [Hz] (with quadruple antiperiodic cell)** *full solution*
1	-	244,18
		245,45
2	245,87	248,21
		249,89
3	-	286,67
		287,19
4	**312,88**	**312,15**
		312,88
5	336,06	341,26
		343,74
6	**491,67**	**491,12**
		491,14

imal Z-component displacements in the whole structure arise when these supports move entirely in this direction. This leads to the conclusion that the crucial information about AMM's nature of motion could be read only from selective modes.

The vibration velocity response spectra indicate the resonance corresponding to the first Z-directional mode for every beam (Fig. 8). The results for both procedures are almost identical.

That proves the partial results could deliver the approximate solution i.e., in the first design stage or in the optimization process. A smaller model can lower the computation time by four

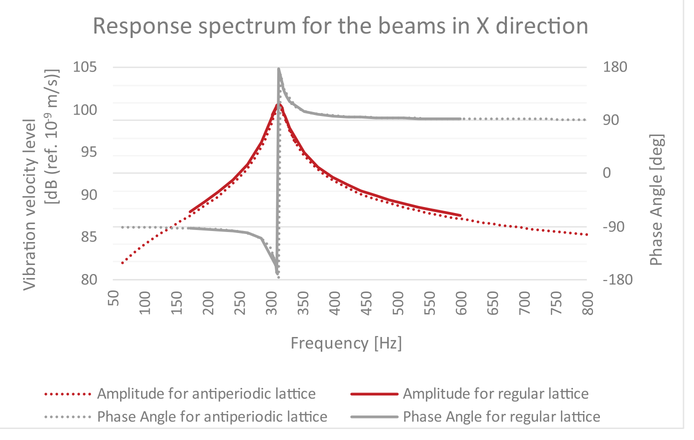

Fig. 8 Response spectra of Z-component vibration velocity level of free end of the beams - a beam turned in the x direction

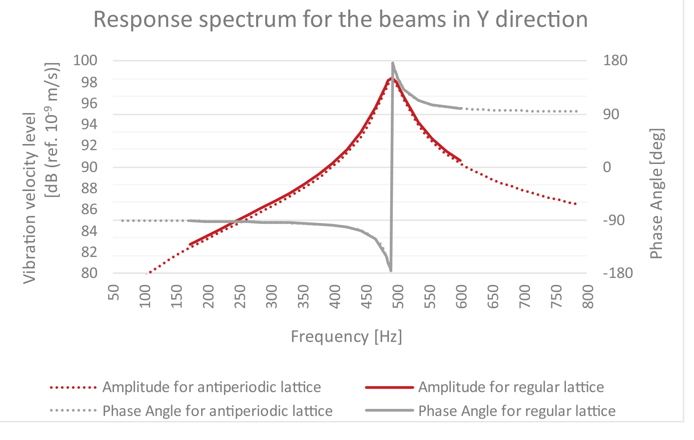

Fig. 9 Response spectra of Z-component vibration velocity level of free end of the beams - a beam turned in the y direction

times or more. Regardless, the antiperiodic lattice procedure could only be used for certain requirements and should not replace the full solution.

5. CONCLUSIONS

The exemplary acoustic plate-type metamaterial demonstrates the method of simulating the periodicity using ANSYS. The procedure allows conducting the periodic or Floquet periodic boundary conditions in both directions simultaneously. The paper offers the antiperiodic lattice approach, which is an alternative to the direct periodic lattice. The presented approach omits the modes requiring the perpendicular deflection of the marginal edges from the result and consequently gives only the partial solution. Nevertheless, when carefully applied, this method yields results almost identical to the full solution, with more than four times less computational time.

LITERATURE

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